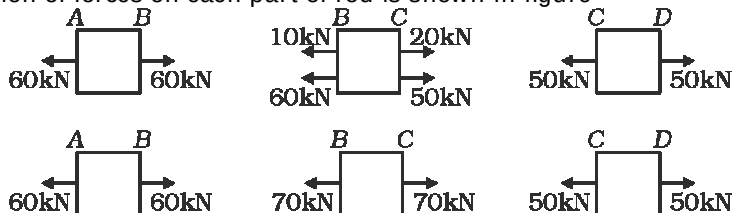


Level - 3

JEE Advanced++ Pattern/Solutions

159. ($1.3\mu m$)

The action of forces on each part of rod is shown in figure



We know that the extension due to external force F is given by $e = \frac{Fl}{AY}$

$$\therefore e_{AB} = \frac{(60 \times 10^3) \times 1.5}{1 \times 2 \times 10^{11}} = 4.5 \times 10^{-7} m$$

$$e_{BC} = \frac{(70 \times 10^3) \times 1}{1 \times 2 \times 10^{11}} = 3.5 \times 10^{-7} m \quad \text{and} \quad e_{CD} = \frac{(50 \times 10^3) \times 2}{1 \times 2 \times 10^{11}} = 5.0 \times 10^{-7} m$$

The total extension $e = e_{AB} + e_{BC} + e_{CD}$

$$= 4.5 \times 10^{-7} + 3.5 \times 10^{-7} + 5.0 \times 10^{-7} = 13 \times 10^{-7} m = 1.3 \mu m$$

160. $\left(\frac{FL}{\pi abY} \right)$

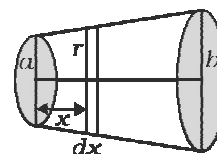
Consider an element of length dx at a distance x as shown.

$$r = a + \frac{b-a}{L}x \Rightarrow dr = \frac{b-a}{L}dx$$

Elongation in the length dx is given by $d(\Delta l) = \frac{Fdx}{\pi r^2 Y}$

$$\text{Total elongation } \Delta l = \int \frac{Fdx}{\pi r^2 Y}$$

$$\Delta l = \int \frac{FLdr}{\pi r^2 Y(b-a)} = \frac{FL}{\pi Y(b-a)} \int_a^b \frac{1}{r^2} dr = \frac{FL}{\pi Y(b-a)} \left[-\frac{1}{r} \right]_a^b = \frac{FL}{\pi abY}$$



161. ($\alpha = 2 \times 10^{-5} / ^\circ C$, $Y = 1.105 \times 10^{11} N/m^2$)

$$\Delta l = l\alpha\Delta\theta$$

$$\Delta l = \Delta l_1 + \Delta l_2$$

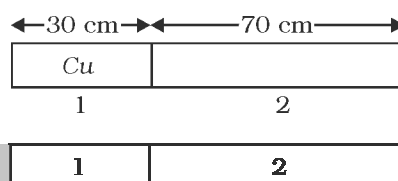
$$(1.91 \times 10^{-3}) = 0.3 \times 1.7 \times 10^{-5} \times 100 + \alpha \times 0.7 \times 100$$

Solving this equation, we get, $\alpha = 2 \times 10^{-5} / ^\circ C$

Compressive force will be same in both rods

$$\text{i.e. } F_1 = F_2$$

$$\begin{aligned} \text{or } Y_1 \times \left(\frac{\Delta l_1}{l_1} \right) \times A &= Y_2 \times \left(\frac{\Delta l_2}{l_2} \right) \times A \quad \text{or} \quad Y_1 \times \alpha_1 \times \Delta\theta = Y_2 \times \alpha_2 \times \Delta\theta \quad \therefore Y_2 = \frac{\alpha_1 Y_1}{\alpha_2} = \frac{\alpha_{Cu} Y_{Cu}}{\alpha_2} \\ &= \frac{(1.7 \times 10^{-5})(1.3 \times 10^{11})}{(2.0 \times 10^{-5})} = 1.105 \times 10^{11} N/m^2 \end{aligned}$$



162. $\left(\frac{2}{3}\right)$

The concrete and iron sections are subjected to the same strain. If σ_c and σ_i denotes compressive stresses, A_c and A_i cross-section area and Y_c and Y_i Young's modulus respectively.

$$\frac{\sigma_c}{Y_c} = \frac{\sigma_i}{Y_i}$$

Let x fraction of load F be shared by concrete; then we have

$$\frac{x F}{A_c Y_c} = \frac{F - x F}{Y_i A_i} \quad \text{or} \quad x = \frac{A_c Y_c}{A_i Y_i + A_c Y_c}$$

On substituting $Y_c = \frac{1}{10} Y_i$, $A_i = \frac{1}{20} A_c$, we get

$$x = \frac{A_c Y_c}{\frac{1}{20} A_c \times 10 Y_c + A_c Y_c} = \frac{2}{3}$$

163. $\left(P = P_0 + B \ln \left(1 - \frac{\rho_0 g h}{B}\right)\right)$

In a static fluid, the pressure variation is given by

$$\frac{dP}{dh} = -\rho g \quad \dots (i)$$

The bulk modulus is defined as

$$B = -V \frac{dP}{dV} \quad \dots (ii)$$

Consider a sample of the fluid having mass M , then its volume

$$V = \frac{M}{\rho} \quad \Rightarrow \quad dV = -\frac{M}{\rho^2} d\rho \quad \dots (iii)$$

From Eqs. (i), (ii) and (iii)

$$\frac{B d\rho}{\rho} = \rho g dh \quad \Rightarrow \quad \int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = \frac{g}{B} \int_0^h dh \quad \Rightarrow \quad \frac{1}{\rho_0} - \frac{1}{\rho} = \frac{gh}{B} \quad \dots (iv)$$

$$\text{Now, } dP = -\frac{B dV}{V} = B \frac{d\rho}{\rho} \quad \Rightarrow \quad \int_{P_0}^P dP = B \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} \quad \Rightarrow \quad P - P_0 = B \ln \frac{\rho}{\rho_0} \quad \dots (v)$$

$$\text{From Eq. (iv)} \quad 1 - \frac{\rho_0}{\rho} = \frac{\rho_0 g h}{B} \quad \therefore \quad \ln \frac{\rho}{\rho_0} = \ln \left(1 - \frac{\rho_0 g h}{B}\right) \quad \Rightarrow \quad P = P_0 + B \ln \left(1 - \frac{\rho_0 g h}{B}\right)$$

164. $\left(\frac{P}{Y}(1 - 2\nu)\right)$

When the rod is not compressed, its density is $\rho_1 = \frac{m}{V_1}$ where $V_1 = \pi r^2 l$.

The density of compressed rod is $\rho_2 = \frac{m}{V_2}$, where

$$V_2 = \pi(r + \Delta r)^2(l - \Delta l)$$

The change in density of rod is

$$\Delta \rho = \rho_2 - \rho_1 = m \left(\frac{1}{V_2} - \frac{1}{V_1} \right) = \frac{m \Delta V}{V_2 V_1}$$

As the compression in a solid is negligibly small, we can safely assume $V_2 V_1 = V_1^2$.

Hence the relative change in density, $\frac{\Delta \rho}{\rho} = \frac{\Delta V}{V_1}$.

$$\begin{aligned} \text{Now } \Delta V &= \pi r^2 l - \pi(r + \Delta r)^2(l - \Delta l) = \pi r^2 l \left(\frac{\Delta l}{l} - \frac{2\Delta r}{r} \right) \\ &\approx V_1 \frac{\Delta l}{l} (1 - 2\nu) \quad \left[\text{as } \nu = \frac{\Delta r / r}{\Delta l / l} \right] \end{aligned}$$

$$\text{Hence } \frac{\Delta \rho}{\rho_1} = \frac{\Delta V}{V_1} = \frac{\Delta l}{l} (1 - 2\nu)$$

$$\text{From Hooke's law, we have } \frac{\Delta l}{l} = \frac{P}{Y} \quad \text{So} \quad \frac{\Delta \rho}{\rho_1} = \frac{P}{Y} (1 - 2\nu)$$